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Stabilization and tracking control for a class of nonlinear systems

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ABSTRACT

This paper discusses stabilization and tracking control using linear matrix inequalities for a class of systems with Lipschitz nonlinearities. A nonlinear state feedback stabilization control is proposed for systems containing a more general case of Lipschitz nonlinearity. The main objective of the present study is to provide, for multi-input multi-output nonlinear systems, a tracking control approach based on nonlinear state feedback, which guarantees global asymptotic output and state tracking with zero tracking error in the steady state. Further, the tracking control is formulated for optimal disturbance rejection, using L_2 gain reduction based performance criteria. The proposed methodologies are illustrated herein using two simulation examples of chaotic and unstable dynamical systems.

1. Introduction

Stabilization and tracking are two of the most important issues currently under consideration by researchers in linear and nonlinear control theory. The former addresses the convergence of system states to the origin or a bounded region containing the origin [1,2]. The latter has two categories, output tracking and state tracking. Both problems deal with the stabilization of system outputs or states to any reference output or desired state (especially an equilibrium point) [3–6]. The stabilization problem is the basic one, and has been extensively studied for both linear and nonlinear systems. Whereas tracking control theory for linear systems is well established in the field [7]; for nonlinear systems, the controller design is a nontrivial problem, and its theory is still being developed. For uncertain, unstable nonlinear systems, tracking control objectives are more difficult to achieve. Indeed, due to the underlying complexity of nonlinear systems [8], many problems remain unsolved to date, despite the development of control laws to address issues such as performance, disturbance rejection and robustness for local or global stabilization and tracking.

Recently, the control community has focused on design and analysis of controllers for Lipschitzian nonlinear systems. In fact, a major class of nonlinear systems satisfies the Lipschitz condition either globally or locally. Moreover, incorporation of the Lipschitz condition into a linear matrix inequality (LMI) offers a tractable formulation for an efficient solution. Thus, many strategies for observer design have been developed for such systems [9,10]. LMI-based linear state feedback, formulated by means of quadratic Lyapunov function and L_2 gain reduction, has been used extensively in addressing stabilization and ensuring performance, robustness, actuator fault tolerance and disturbance attenuation [11–16]. Such techniques are based on a nonlinearity assumption, say, $f(x)$ satisfying $f(x) = 0$ at $x = 0$. Although this makes problem handling easier, the issue of stabilization of classes of systems not verifying this assumption remains.

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Synthesis of tracking controllers for Lipschitzian nonlinear systems is an interesting and important subject that, unfortunately, has received entirely inadequate attention thus far. For linear systems, modifying existing stabilization techniques for tracking control by incorporation of a feed-forward controller is very straightforward. For nonlinear systems, this feed-forward controller must be correspondingly nonlinear or adaptive, which fact complicates its synthesis. Traditionally, researchers seeking to resolve this nontrivial problem have applied neural network, fuzzy control and adaptive control based strategies [17–20], which, however, have their own disadvantages. Specifically, these strategies are computationally complex, amounting to selection or tuning of a finite number of parameters. Most of their applications (for instance, [18–20]) are limited to single-input single-output (SISO) nonlinear systems and based on complex design procedures.

In this paper, we propose a nonlinear state feedback control for stabilization of a class of Lipschitzian nonlinear systems, which strategy modifies the traditional linear state feedback theory. The modified control law ensures asymptotic stability of the systems even if the nonlinear part $f(0) \neq 0$. The main objective of this study is to formulate a new approach for asymptotic tracking control of multi-input multi-output (MIMO) nonlinear systems that utilizes LMI-based state feedback. The idea is to calculate the desired state values for an output reference and to create an equilibrium point for the system at those values. By ensuring the global asymptotic stability of a newly created equilibrium point, the desired output tracking is achieved. Hence, the proposed tracking control ensures both state tracking and output tracking for a specific class of systems. LMI conditions for control laws are developed by means of quadratic Lyapunov function and the Lipschitz condition [9,21]. This technique is further modified for optimal disturbance rejection using L_2 gain reduction based performance criteria. These control strategies are, though nonlinear, computationally simple, easy to design and implement and flexible due to utilization of LMIs. The reported schemes in the present study were applied to chaotic and unstable simulation examples, and the results are offered herein.

We use standard notations. The L_2 gain from d to z is defined as $\sup_{\|d\|_2 \neq 0} \|z\|_2/\|d\|_2$, where $\|.\|_2 = \sqrt{\int_0^\infty \|.\|^2 \mathrm{d}t}$ denotes the L_2 norm and $\|.\|$ is the Euclidean norm. For x_i with the ith diagonal entry and $i = 1, 2, \ldots, n$, $\mathrm{diag}(x_1, x_2, \ldots, x_n)$ denotes a diagonal matrix.

This paper is organized as follows. Section 2 treats the LMI-based stabilization for nonlinear systems. Section 3 discusses the controller design for asymptotic tracking control and its further modification for disturbance rejection. Section 4 illustrates the simulation results with two simulation examples including chaotic Chua's circuit. Section 5 draws conclusions.

2. State feedback stabilization

Consider the following class of systems with Lipschitz nonlinearity.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) + Ax + Bu + d, \quad x(0) = x_0, \tag{1}$$

$$y = Cx, \tag{2}$$

where $x \in R^n, y \in R^n, u \in R^n$ and $d \in R^n$ represent the state, the output, the input to the system, and the disturbance, respectively. The nonlinearity $f(x) \in R^n$ is a time-varying vector. The matrices $A \in R^{n\times n}, B \in R^{n\times n}$ and $C \in R^{n\times n}$ are constant square matrices, and B and C are invertible. The initial condition is $x(0) = x_0$.

Assumption 2.1. The function $f(x)$ is Lipschitz for all $x \in R^n$ and $\bar{x} \in R^n$, and satisfies

$$\|f(x) - f(\bar{x})\| \leq \|L(x - \bar{x})\|, \tag{3}$$

where $L \in R^{n\times n}$ is a Lipschitz constant matrix. Traditionally, it is assumed that the function $f(x)$ vanishes at $x = 0$ [11–16]. We modify the linear state feedback control to deal with the systems that do not follow this characteristic, and derive a sufficient condition for global asymptotic stabilization. Note that the matrix C is not necessarily invertible for the stabilization problem. We will consider C as an invertible matrix only for the tracking problem, which is addressed in Section 3.

Remark 2.1. The matrices B and C in (1)–(2) are assumed to be square in this paper. In fact, our work deals also with the situation in which these matrices are non-square and have full row rank. Such matrices satisfy $BB_r^{-1} = I$ and $CC_r^{-1} = I$, where B_r^{-1} and C_r^{-1} represent the right inverses of B and C, respectively. An example is provided in the simulation results (Section 4).

Assumption 2.2. Assume that the disturbance $d = 0$.

The proposed nonlinear state feedback control for stabilization is given by

$$u = Fx - B^{-1}f(0), \tag{4}$$

where F is the state feedback gain. The control law (4) has the additional term $B^{-1}f(0)$, unlike the conventional state feedback, which is essential in order to deal with systems having $f(0) \neq 0$. Now we provide a sufficient stabilization condition for system (1) using the proposed control law.

Theorem 2.1. *Suppose that system (1) satisfies the Assumptions 2.1 and 2.2. The control law (4) ensures the global asymptotic stability of the system states x if the following LMIs are satisfied.*

$$Q = Q^T > 0, \quad \Psi = \begin{bmatrix} QA^T + AQ + M^T B^T + BM & I & QL^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{5}$$

Moreover F can be found by $F = MQ^{-1}$.

Proof. From Assumption 2.2, using $d = 0$ and substituting (4) into (1), we obtain

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) - f(0) + (A + BF)x, \quad x(0) = x_0. \tag{6}$$

Consider the following quadratic Lyapunov function candidate.

$$V = x^T P x, \quad \text{with } P = P^T > 0. \tag{7}$$

The derivative of (7) along (6) is given by

$$\dot{V} = x^T(A^T + F^T B^T)Px + x^T P(A + BF)x + (f(x) - f(0))^T Px + x^T P(f(x) - f(0)). \tag{8}$$

The inequality (3) can be written as

$$(f(x) - f(0))^T I (f(x) - f(0)) \le x^T L^T L x. \tag{9}$$

From (8) and (9), we have

$$\begin{aligned} \dot{V} \le & \left\{ x^T(A^T + F^T B^T)Px + x^T P(A + BF)x + (f(x) - f(0))^T Px \right. \\ & \left. + x^T P(f(x) - f(0)) - (f(x) - f(0))^T I (f(x) - f(0)) + x^T L^T L x \right\}, \end{aligned} \tag{10}$$

which, further, can be written

$$\dot{V} \le X^T \Omega X, \tag{11}$$

$$\text{where } X = \begin{bmatrix} x^T & (f(x) - f(0))^T \end{bmatrix}^T, \tag{12}$$

$$\text{and} \quad \Omega = \begin{bmatrix} (A^T + F^T B^T)P + P(A + BF) + L^T L & P \\ * & -I \end{bmatrix} < 0, \tag{13}$$

because for asymptotic stability $\dot{V} < 0$. Applying the Schur complement [22–24], we obtain the matrix inequality

$$\begin{bmatrix} (A^T + F^T B^T)P + P(A + BF) & P & L^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{14}$$

Now, applying congruence transform by pre- and post-multiplying the inequality (14) by $\mathrm{diag}(P^{-1}, I, I)$ and then using $P^{-1} = Q > 0$ and $M = FQ$, we obtain the LMIs in (5), which completes the proof. □

3. Tracking control using state feedback

To provide a control law for tracking, we once again consider system (1). The proposed nonlinear state feedback control law, accordingly, is given by

$$u = Fx + u_r, \tag{15}$$

with

$$u_r = -Fx_r - B^{-1}Ax_r - B^{-1}f(x_r), \quad x_r = C^{-1}r, \tag{16}$$

where $x_r \in R^n$ and $r \in R^n$ are the reference state and reference signal for output tracking. For state tracking, x_r is provided by the user, in that case, we can exempt $x_r = C^{-1}r$ from control law (15)–(16).

Remark 3.1. The control law (15)–(16) is computationally simple because it has no feed-forward adaptive tracking controller. Moreover, the term u_r remains constant for a specific reference r, so its computation is required only when a new reference is applied.

Now we develop an LMI-based sufficient condition for determining F in (15)–(16), which guarantees the asymptotic output tracking.

Theorem 3.1. *Suppose that system (1)–(2) satisfies Assumptions 2.1 and 2.2. If there exists a symmetric matrix Q such that the LMIs*

$$Q > 0 \quad \text{and} \quad \Psi < 0, \tag{17}$$

are satisfied with Ψ given by (5), control law (15)–(16) ensures that

(i) *State x converges asymptotically to x_r;*
(ii) *Output y converges asymptotically to r.*

Proof. Using (15)–(16) in (1), we obtain

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) - f(x_r) + A(x - x_r) + BF(x - x_r) + d, \tag{18}$$

$$x(0) = x_0, \qquad x_r = C^{-1}r. \tag{19}$$

Taking $z = x - x_r$ and $\dot{z} = \dot{x}$ (because x_r is constant for a desired constant reference r), and using Assumption 2.2, we can transform (18) into

$$\frac{\mathrm{d}z}{\mathrm{d}t} = f(x) - f(x_r) + (A + BF)z, \quad z(0) = x_0 - x_r. \tag{20}$$

Typically, one can assume that the initial condition $x(0) = x_0$ is zero for the output tracking problem, though we are not taking this assumption in the present case. We now prove the asymptotic stability of state z in (20) by considering the quadratic Lyapunov function candidate

$$V = z^T P z, \quad \text{with } P = P^T > 0. \tag{21}$$

Using the same procedure in Section 2, we obtain the inequality

$$\dot{V} \le Z^T \Omega Z < 0, \tag{22}$$

where

$$Z = \begin{bmatrix} z^T & (f(x) - f(x_r))^T \end{bmatrix}^T. \tag{23}$$

It was seen in the proof of Theorem 2.1 that $\Omega < 0$ leads to $Q = Q^T > 0$ and $\Psi < 0$, which proves the asymptotic stability of (20). As z converges to zero asymptotically, state x converges to x_r, which demonstrates the validity of statement (i) in Theorem 3.1. For this reason, the output in the steady state becomes $y = Cx_r$. According to (19), $x_r = C^{-1}r$, which shows that the steady state output is $y = r$. This completes the proof of statement (ii) in Theorem 3.1. □

Remark 3.2. The proposed tracking control strategy is based on LMIs in contrast to other methodologies for Lipschitz nonlinear systems [17–20], which makes the computation of controller parameters easier, due to available sophisticated LMI-routines. Moreover, incorporation of other performance objectives like robustness and time-domain performance are not problematic due to flexibility of LMIs. Computational complexity, tuning of parameters and restricted applicability to SISO systems are also limitations of traditional techniques.

Thus far, we have derived a sufficient condition for the tracking control of systems (1)–(2) by assuming zero disturbances. Now we address the issue of robust output tracking against disturbances by minimizing the L_2 gain from disturbance d to the error $e = r - y$ under the following assumptions.

Assumption 3.1. The L_2 norm of disturbance d is bounded.

Assumption 3.2. Reference signal $r = 0$ at time $t = 0$ (by this we mean that a constant reference r is applied at any time $t > 0$) and $x(0) = x_0 = 0$. This further implies that $z(0) = 0$.

Theorem 3.2. *Suppose that system (1)–(2) satisfies Assumptions 2.1, 3.1 and 3.2. Consider the optimization problem*

$$\min \gamma$$

such that

$$\gamma > 0, \qquad Q = Q^T > 0, \tag{24}$$

$$\text{and} \quad \Phi = \begin{bmatrix} QA^T + AQ + M^T B^T + BM & I & I & QL^T & QC^T \\ * & -I & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0. \tag{25}$$

Accordingly, the nonlinear control law (15)–(16) with $F = MQ^{-1}$ ensures that

(i) *State x and output y asymptotically converge to x_r and r, respectively, if the disturbance $d = 0$;*
(ii) *Output error $e = r - y$ satisfies $\|e\|_2^2 < \gamma^2\|d\|_2^2$, if disturbance $d \neq 0$.*

Proof. Taking $z = x - x_r$, $\dot{z} = \dot{x}$ and $e = r - y = -Cz$, and using Assumption 3.2, the system (18)–(19) is rewritten as

$$\frac{\mathrm{d}z}{\mathrm{d}t} = f(x) - f(x_r) + (A + BF)z + d, \quad z(0) = 0, \tag{26}$$

$$e = -Cz. \tag{27}$$

Consider the quadratic Lyapunov function

$$V = z^T P\gamma z, \quad \text{with } P = P^T > 0, \text{ and } \gamma > 0. \tag{28}$$

Defining

$$J = (\dot{V} + e^T e - \gamma^2 d^T d)/\gamma < 0, \tag{29}$$

and integrating from 0 to $T \to \infty$, we obtain

$$\int_0^T J\mathrm{d}t = (V(T) - V(0))/\gamma + \frac{1}{\gamma}\int_0^T e^T e\mathrm{d}t - \gamma\int_0^T d^T d < 0, \tag{30}$$

which implies the following.

(a) If $d = 0$, then (29) shows $\dot{V} + z^T z < 0$, that is $\dot{V} < 0$. Hence, the system (26)–(27) is asymptotically stable and z and e converge to zero asymptotically. This ensures that x and y converge asymptotically to x_r and r, respectively.
(b) Given $z(0) = 0$, $V(0) = 0$. Also noting that $V(T) > 0$, (30) ensures $\|e\|_2^2 < \gamma^2\|d\|_2^2$.

Taking the derivative of the Lyapunov function of (28) along (26)–(27) and substituting the resultant into (29), we have

$$\begin{aligned} J = \big\{&z^T(A^T + F^T B^T)Pz + z^T P(A + BF)z + (f(x) - f(x_r))^T Pz \\ &+ z^T P(f(x) - f(x_r)) + d^T Pz + z^T Pd + (1/\gamma)z^T C^T Cz - \gamma d^T d\big\} < 0. \end{aligned} \tag{31}$$

Using inequality (3), we obtain

$$\begin{aligned} &\big\{z^T(A^T + F^T B^T)Pz + z^T P(A + BF)z + (f(x) - f(x_r))^T Pz + z^T P(f(x) - f(x_r)) + (1/\gamma)z^T C^T Cz - \gamma d^T d + d^T Pz \\ &\quad + z^T Pd - (f(x) - f(x_r))^T I(f(x) - f(x_r)) + x^T L^T Lx\big\} < 0, \end{aligned} \tag{32}$$

which, further, can be written as

$$Z^T \Pi Z < 0, \tag{33}$$

where

$$Z = \begin{bmatrix} z^T & (f(x) - f(x_r))^T & d^T \end{bmatrix}^T, \tag{34}$$

and

$$\Pi = \begin{bmatrix} (A^T + F^T B^T)P + P(A + BF) + L^T L + (1/\gamma)C^T C & P & P \\ * & -I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0. \tag{35}$$

Using the Schur complement, we get

$$\begin{bmatrix} (A^T + F^T B^T)P + P(A + BF) & P & P & L^T & C^T \\ * & -I & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0. \tag{36}$$

Now applying the congruence transform by pre- and post-multiplying the matrix inequality (36) by $\mathrm{diag}(P^{-1}, I, I, I, I)$, and taking $P^{-1} = Q > 0$ and $M = FQ$ (see [22–24]), we obtain the LMIs given by Theorem 3.2, which completes the proof. □

Remark 3.3. Conventional robust control techniques are based on the minimization of L_2 gain from reference signal r (contained in exogenous input) to output error e [7,16], which does not nullify errors even if disturbance $d = 0$ (see also [25–27]). In the present scenario, Theorem 3.1 provides a methodology that nullifies the tracking error in the steady state, and Theorem 3.2 modifies its results by which the minimization of L_2 gain is required from exogenous signal d

(which does not contain r) to error e. This distinction makes our results less conservative than the traditional methodologies. Moreover, the proposed computationally simpler tracking control approach is applicable to more general cases of Lipschitz nonlinearity with $f(0) \neq 0$, due to the exceptional structure of the controller (15)–(16).

Remark 3.4. The present work on tracking control is useful for both output tracking and state tracking. By specifying any desired x_r, rather than using $x_r = C^{-1}r$, in the control law, one can achieve the state tracking.

Remark 3.5. The proposed tracking control strategy given by Theorems 3.1 and 3.2 can be modified for a continuously differentiable time-varying reference signal r. For this purpose, the control law (15)–(16) can be modified as

$$u = Fx + u_r, \tag{37}$$

with

$$u_r = -Fx_r - B^{-1}Ax_r - B^{-1}f(x_r) + B^{-1}\dot{x}_r, \tag{38}$$

$$x_r = C^{-1}r, \qquad \dot{x}_r = C^{-1}\dot{r}, \tag{39}$$

where $\dot{r}$ and $\dot{x}_r$ are the derivatives of r and x_r, respectively. If the reference signal r is arbitrary such that $\dot{r}$ is unknown, then we can replace $\dot{r}$ with its backward difference approximation. It can be easily verified that the LMI conditions developed in Theorems 3.1 and 3.2 are applicable for the modified control law (37)–(39).

Remark 3.6. A wide class of nonlinearities satisfies the Lipschitz condition locally. It is stated in [9] that if the region of operation of a plant in terms of states is known, a local controller can be designed. For the output tracking problem, we know the range of reference signal r that can be used to determine the range of reference states utilizing $x_r = C^{-1}r$. This provides a useful information for selecting the range of states.

It is often observed in practice that the optimization problem given in Theorem 3.2 computes undesirable higher or lower entries of the gain matrix F [15,22–24,28]. This issue can be resolved by solving the LMIs of (24)–(25), for feasibility, with a desirably lower selection of γ [15,24]. The control laws (4), (15)–(16) and (37)–(39) are extendable if matrices B and C are non-square, by replacing B^{-1} and C^{-1} with B_r^{-1} and C_r^{-1} (if they exist). This situation is explained by way of a simulation example in Section 4. Presently we are dealing with stabilization, tracking control, and disturbance rejection. A number of issues, such as time-domain performance and robustness against internal perturbations, are held over for upcoming studies.

4. Simulation results

To show the effectiveness of the proposed methodologies, two numerical examples are presented in this section.

Example 1. We select a Chua's circuit, to demonstrate the applicability of the proposed scheme on chaotic physical systems, as chaos control is receiving substantial interest of scientific community [29,30]. The dynamics are given by

$$A = \begin{bmatrix} -2.548 & 9.1 & 0 \\ 1 & -1 & 1 \\ 0 & -14.2 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{40}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad f(x) = \frac{1}{2}\begin{bmatrix} |x_1 + a_1| - |x_1 - a_2| \\ 0 \\ 0 \end{bmatrix}, \tag{41}$$

where $a_1 = 1$ and $a_2 = 1.1$. It is worth mentioning that the parameters a_1 and a_2 are taken different, indicating the fact that physical components of an electrical circuit cannot be identical at all. Due to this reason, $f(0) = \begin{bmatrix} -0.05 & 0 & 0 \end{bmatrix}^T \neq 0$. We can select L as

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{42}$$

Fig. 1 shows the phase portrait of the chaotic Chua's circuit. The initial condition is $x_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. The solution of Theorem 3.2 yields F given by

$$F = \begin{bmatrix} -45.51 & -4.91 & -0.03 \\ -7.39 & -33.132 & 6.655 \\ 0.0455 & 6.545 & -34.13 \end{bmatrix}. \tag{43}$$

Fig. 2 shows the time responses of the system states. The control law (15)–(16) is applied at $t = 100$, and the modified control law (37)–(39) for the time-varying reference signal is applied at $t = 200$. The reference signal is given by

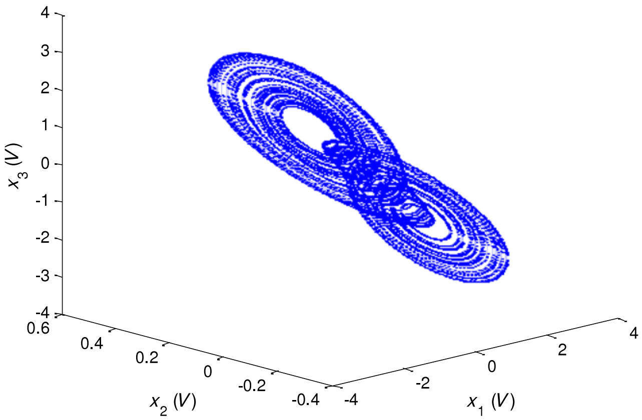

Fig. 1. Phase portrait of chaotic Chua's circuit (40)–(41).

$$x_r = \begin{cases} [0 \quad 0 \quad 0]^T, & 100 \le t < 125, \\ [1 \ -0.2 \ 1]^T, & 125 \le t < 150, \\ [-1 \ 0.2 \ -1]^T, & 150 \le t < 175, \\ [0\ 0\ 0]^T, & 175 \le t < 200, \\ [r_1\ r_2\ r_3]^T, & 200 \le t < 300. \end{cases} \tag{44}$$

where

$$r_1 = 2\sin(0.3(t-200)), \qquad r_2 = 0.4\cos(0.2(t-200)), \qquad r_3 = -3\sin(0.25(t-200)). \tag{45}$$

It is clearly seen that all the states are rapidly tracking the reference signal with reasonable time-domain performance.

Example 2. Consider an unstable nonlinear system (1) described by

$$A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.1 & 0.1 & -0.5 \\ 0.3 & -0.4 & -0.3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.012 & 0.013 & 0.014 & 0.016 \\ 0.01 & 0.014 & 0.01 & 0 \\ 0.013 & 0.017 & 0.018 & 0.011 \end{bmatrix}, \tag{46}$$

$$C = \begin{bmatrix} 1.5 & 2 & 1.25 \\ 0.84 & 0.5 & 0.2 \end{bmatrix}, \quad \text{and} \quad f(x) = \begin{bmatrix} 0.2\cos x_1 \\ 0.3\sqrt{x_2^2+5} \\ 0.4\sin x_3 \end{bmatrix}. \tag{47}$$

The Lipschitz matrix is given by

$$L = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}. \tag{48}$$

Using Theorem 2.1 or 3.1, the following value of F is obtained.

$$F = \begin{bmatrix} -75.12 & -91.86 & 180.88 \\ -56.57 & -59.01 & 63.46 \\ 83.69 & 105.47 & -207.32 \\ -10.93 & 53.45 & -5.31 \end{bmatrix}. \tag{49}$$

Fig. 3 shows the results for stabilization of three states using the control law given by Theorem 2.1. The initial condition is taken as $x(0) = \begin{bmatrix} 10 & 4 & -6 \end{bmatrix}^T$. All three states are converging to zero. The output responses using Theorem 3.1 are shown in Fig. 4(a) and (b). Although the performance of the controller for overshoot and undershoot is not good, the reference tracking objective is successfully achieved.

For robustness, we select $\gamma = 0.6$ and obtain the following controller gain F by solving the LMIs of Theorem 3.2.

$$F = \begin{bmatrix} -773.4 & -841.24 & -83.68 \\ -433.87 & -545.11 & -100.03 \\ 896.28 & 896.5 & -49.28 \\ -162.24 & 38.56 & 14.06 \end{bmatrix}. \tag{50}$$

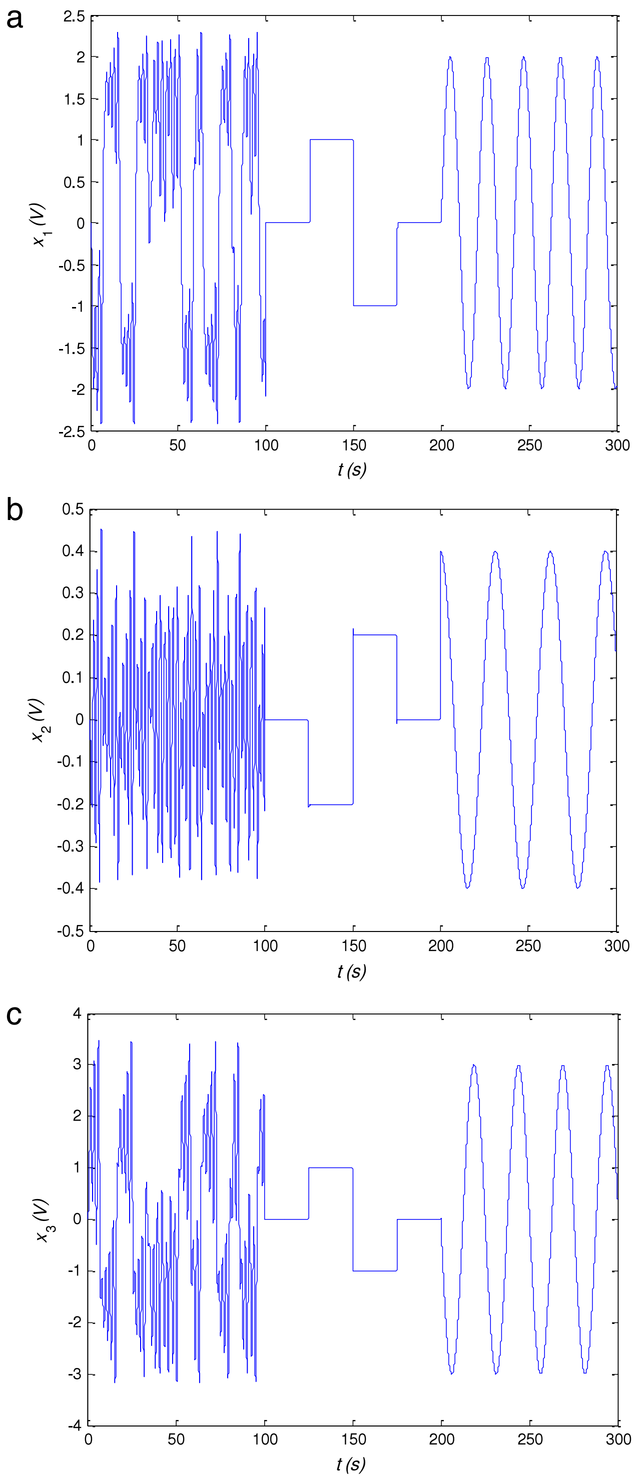

Fig. 2. Time responses of Chua's circuit states using the proposed tracking control: (a) state x_1, (b) state x_2, (c) state x_3.

The disturbance vector is selected as

$$d = 0.4\left[\sin(1.5t) \quad \sin(1.6t) \quad \sin(1.2t)\right]^T. \tag{51}$$

The output responses using Theorems 3.1 and 3.2 under disturbances are shown in Fig. 5(a) and (b). The results by Theorem 3.2 are robust against disturbances and show reasonable time-domain performance as well.

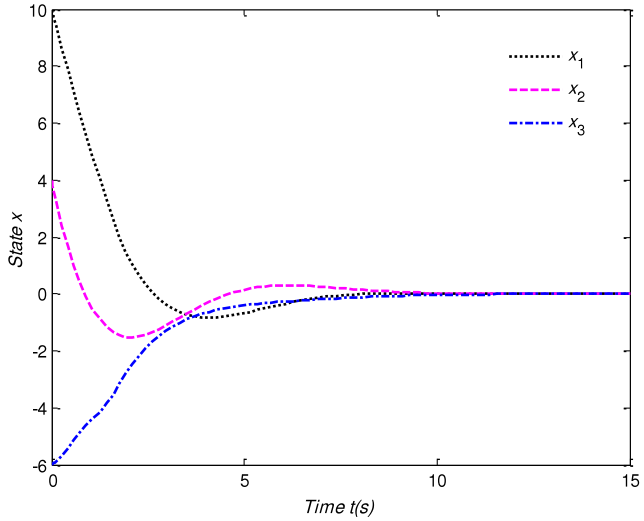

Fig. 3. Stabilization of the system in Example 2 using the control law in Theorem 2.1.

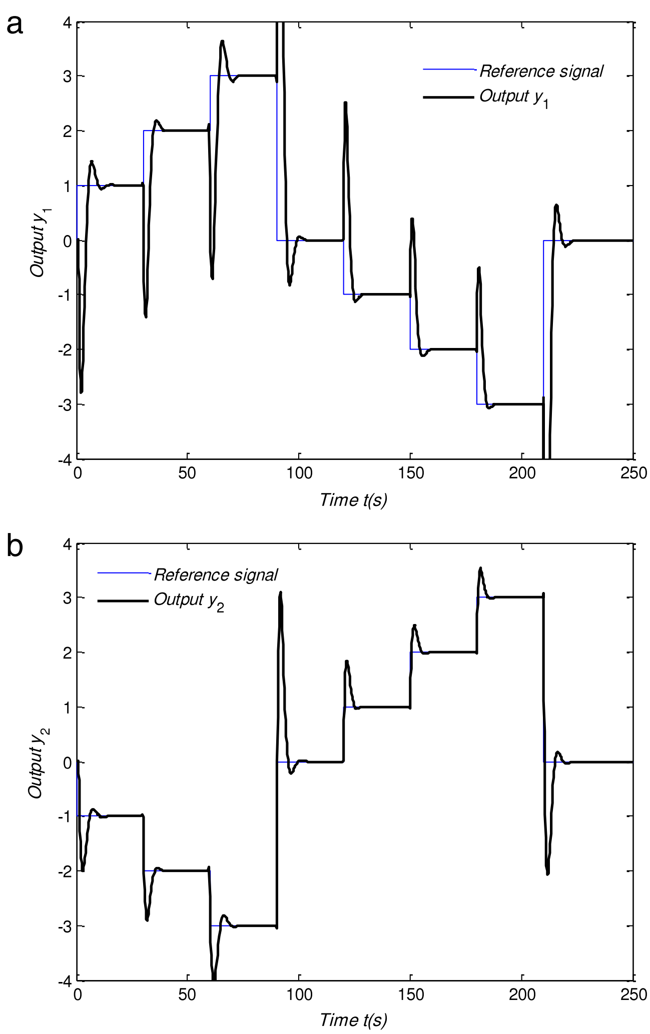

Fig. 4. Reference tracking in Example 2 using the control law in Theorem 3.1: (a) tracking of output y_1, (b) tracking of output y_2.

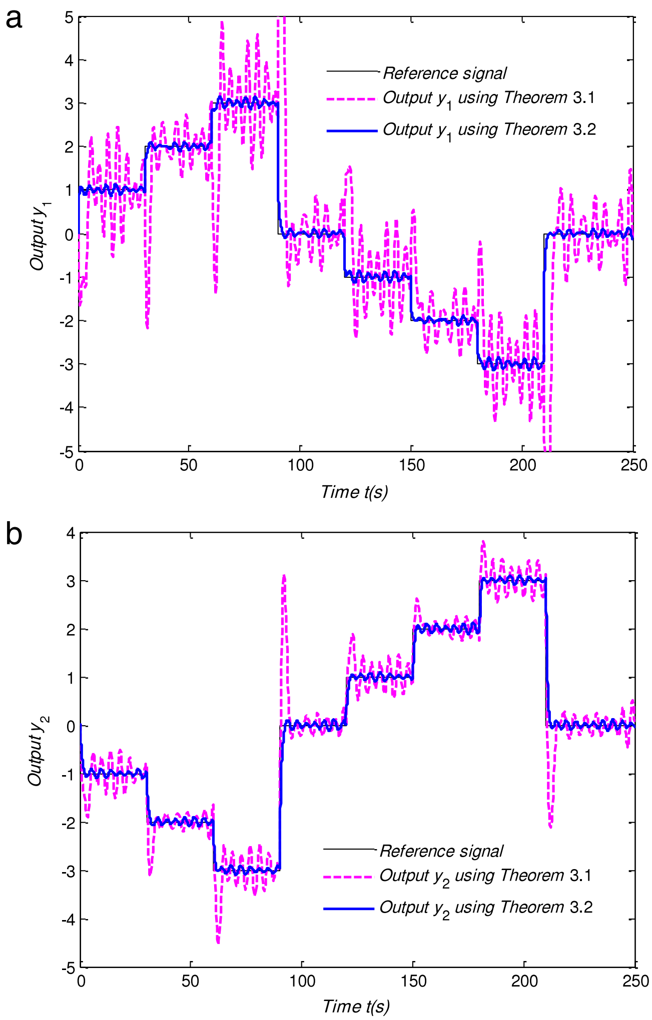

Fig. 5. Robust reference tracking in Example 2 under disturbances using the control law of Theorem 3.2: (a) tracking of output y_1, (b) tracking of output y_2.

5. Conclusions

This paper discusses asymptotic stabilization and tracking control utilizing LMIs for a class of nonlinear Lipschitzian systems. The tracking control strategy was based on achieving specific state values corresponding to a desired output reference. Owing to this feature, this methodology is found also to be useful for state tracking, which is often required in physical systems. Additionally, the output tracking control was modified for disturbance attenuation that, unlike other conservative methodologies, does not use L_2 norm reduction from reference signal to error. Internal robustness and time-domain performance are issues to be addressed in future works.

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